APPLECROSS
SENIOR HIGH SCHOOL

YEAR 12 MATHEMATICS METHODS UNIT 3

TEST 1
TERM 1, 2022
Test date: Tuesday 8th of March

STUDENT NAME: Solutions

All working must be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks.

	Total	Result	
Section 1	26		%
Section 2	29		
Total	55		

Section 1: Resource – Free

Working time: 25 minutes

Question 1 **[2, 2, 3 = 7 marks]** 3.1.9, , 3.1.9

Determine the derivative of each of the following. Express your answers with positive indices.

a) $y = (3x-3)^3$

$$\frac{dy}{dx} = 3(3x-3)^2(3)$$ ✓ uses chain rule

$$\frac{dy}{dx} = 9(3x-3)^2$$ ✓ answer includes $\frac{dy}{dx}$ or y'

b) $y = \dfrac{4x^3}{2x+1}$ (Do not simplify)

$$\frac{dy}{dx} = \frac{(2x+1)(12x^2) - 4x^3(2)}{(2x+1)^2}$$

✓ uses quotient rule

✓ correct answer includes $\frac{dy}{dx}$ or y'

c) $y = \frac{1}{2}x^2(\sqrt[3]{1-3x})$ (Do not simplify)

$$\frac{dy}{dx} = \frac{1}{2}\left[x^2\left(\frac{1}{3}(1-3x)^{-2/3}(-3)\right) + \sqrt[3]{1-3x}\,(2x)\right]$$

$$= \frac{1}{2}\left[-\cancel{x^2}\sqrt[3]{(1-3x)^2} + 2x\sqrt[3]{1-3x}\right]$$

✓ uses product rule

✓ uses chain rule

✓ correct answer

Question 2 **[3, 3 = 6 marks]** 3.2.1 3.2.2 3.2.3 3.2.6

For each of the following, find $f(x)$, simplifying where possible.

a) $f'(x) = \frac{3x^4+x}{\sqrt{x}}$

$f'(x) = \frac{3x^4}{x^{\frac{1}{2}}} + \frac{x}{x^{\frac{1}{2}}}$

✓ uses correct integral notation

$f'(x) = 3x^{7/2} + x^{1/2} \implies f(x) = \int (3x^{7/2} + x^{1/2})\,dx$

$= \frac{3x^{9/2}}{9/2} + \frac{x^{3/2}}{3/2} + C$

$f(x) = \frac{2x^{9/2}}{3} + \frac{2x^{3/2}}{3} + C$

✓ correct $f(x)$

✓ includes constant

b) $f'(x) = 2(6x-5)^3$

let

$\therefore \quad f(x) = \int 2(6x-5)^3\,dx$ ✓ sets up integral

$= 2\int (6x-5)^3\,dx$

$f(x) = \frac{2(6x-5)^4}{6\times 4} + C$ ✓ integrates $(ax+b)^n$

$f(x) = \frac{(6x-5)^4}{12} + C$ ✓ correct & includes constant

Question 3 [2, 3 = 5 marks] 3.1.10

The cost (in dollars) to make x cans of "Tony's Penguin Food" can be modelled by the function;

$$C(x) = 0.25x^2 - x + 15, \qquad \text{where} \qquad 0 \le x \le 60$$

a) Determine the marginal cost when 40 cans are made.

Marginal Cost $= C'(x)$

$C'(x) = 0.5x - 1$ ✓ correct derivative

$C'(40) = 0.5(40) - 1$

$= 19$ dollars ✓ correct marginal cost including units.

b) Determine the average rate of change in cost when making the first 50 cans.

Average r.o.c $= \dfrac{C(50) - C(0)}{50 - 0}$ ✓ uses r.o.c.

$= \dfrac{(0.25(50)^2 - 50 + 15) - 15}{50}$ ✓ subst.

$= \dfrac{575}{50} = \$11.50/\text{can}$ ✓ correct answer

Question 4 [3 marks]

A side of a cube is measured with 3% error. 3.1.10

Find the approximate percentage error in the surface area of the cube. Using an understanding of increments

$SA_{cube} = A(l) = 6l^2$ $\qquad \delta l = 0.03l$

$A'(l) = 12l$

$\cancel{\delta A \approx \frac{dA}{dl} \times \delta l^2 \approx 12l \times 0.03l \approx \frac{12l \times 0.03}{6l^2}}$

Correct derivative function

$\dfrac{\delta A}{A} \approx \dfrac{\delta A}{A} \approx \dfrac{A'(l) \times \frac{\delta l}{l}}{A}$

✓ States incremental change

$\dfrac{\delta A}{A} \approx \dfrac{12l \times 0.03l}{6l^2}$

✓ uses increments formula

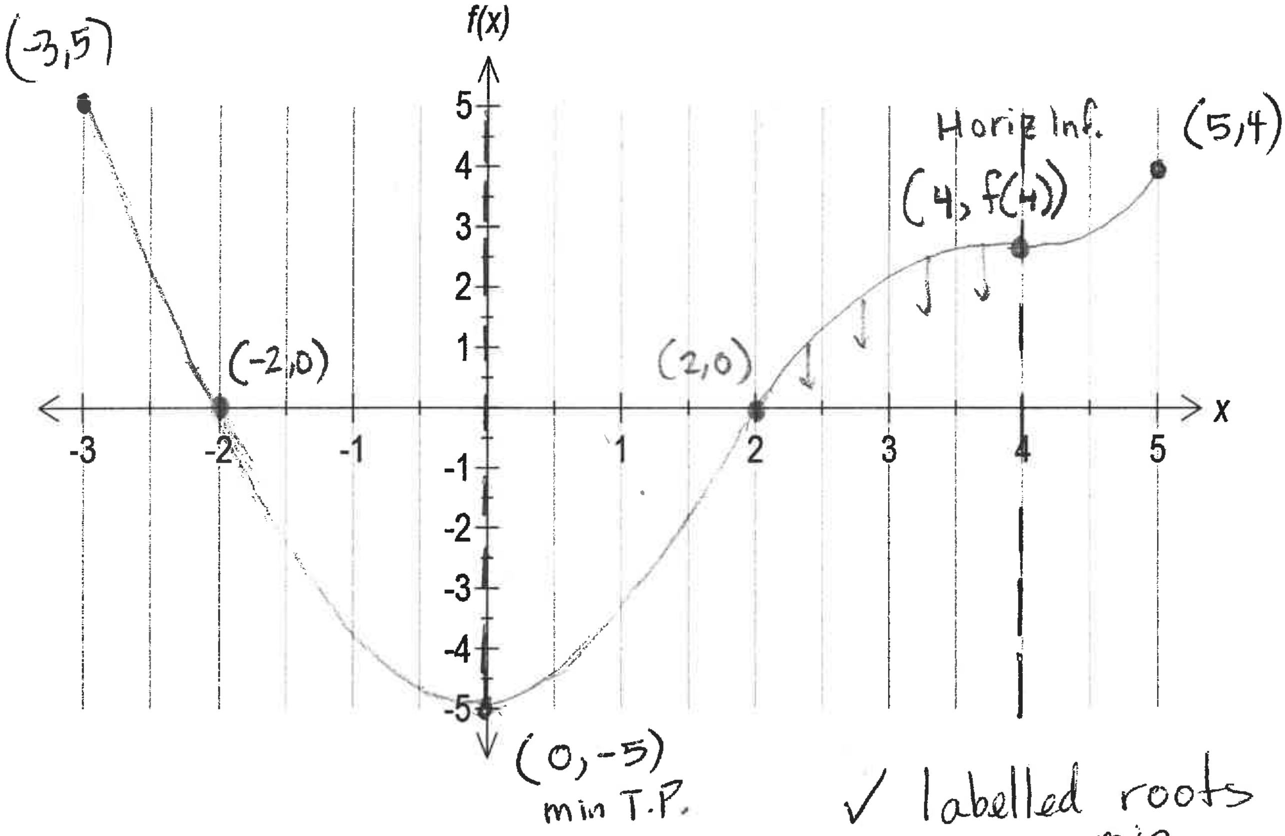
$\dfrac{\delta A}{A} \approx 0.06$

✓ $\therefore$ Approx. % error in Surface Area is 6%.

Question 5 **[5 marks]** 3.1.15

On the axes below, sketch a function $f(x)$ (over the domain $-3 \le x \le 5$) that satisfies all of the following conditions listed.

- $f(-2)=f(2)=0$ ✓
- $f'(0)=f'(4)=0$
- $f''(4)=0$
- $f''(x) < 0$ ONLY when $2 < x < 4$
- $f'(x) < 0$ ONLY when $x < 0$
- The global maximum and minimum of $f(x)$ over this domain are 5 and -5 respectively.
- $f(5)=4$



✓ labelled roots

✓ labelled min T.P. at (0,-5)

✓ Horiz. Infl. at (4, f(4))

✓ global max at (-3,5)

✓ sketched over domain (smooth curve)

END OF SECTION 1

YEAR 12 MATHEMATICS
METHODS UNIT 3

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29'

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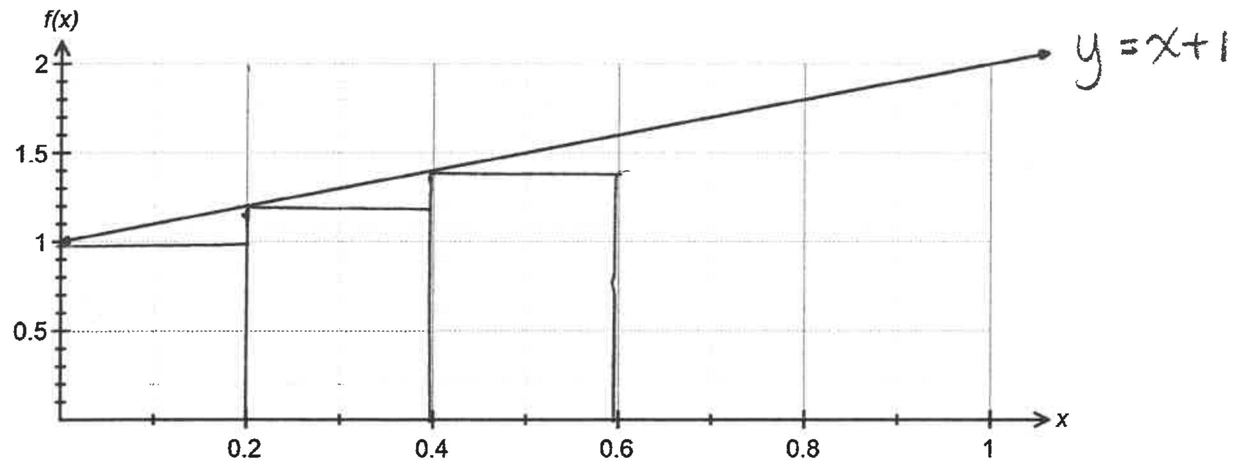
Section 2: Resource – Rich

Working time: ~~25~~ 30 minutes

To be provided by the student:
ClassPad and/or Scientific Calculators
1 sheet of A4–sized paper of notes, double-sided

Question 6 **[1, 2, 2 = 5 marks]** 3.2.10, 3.2.12

Consider the function $f(x)$ drawn below.



a) Draw rectangles on your graph that can be used to underestimate the area under $f(x)$ over the domain $0 \le x \le 0.6$, where $\delta x = 0.2$.

✓ (3 rectangles)

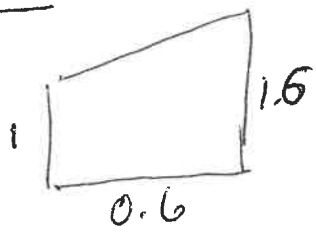
b) Determine the underestimated area of $f(x)$

Underest. Area $\approx 0.2(1 + 1.2 + 1.4) \approx 0.72$ sq.units

✓ uses rectangles ✓ approximation

c) Use the graph of $f(x)$ above to calculate $\int_0^{0.6} f(x)dx$

either approach ok, But Must Show method



✓method ✓ans

$\int_0^{0.6} f(x)dx = \frac{1}{2}\, 0.6(1+1.6) = 0.78$ sq.units

OR $\int_0^{0.6} (x+1)dx = \frac{39}{50}$

Question 7 **[3, 3, 3 = 9 marks]**

Given the curve with the equation $y = \frac{2x^2-1}{3-x^2}$. (3.1.13, 3.1.14)

a) This curve has only one stationary point. Use calculus methods to find the coordinates of this point.

$$y = \frac{2x^2-1}{3-x^2}$$

$$y' = \frac{10x}{(x^2-3)^2}$$ (ClassPad) ✓ first deriv.

sets $y'=0$ & solves ✓

let $y'=0$ $\quad 0 = \frac{10x}{(x^2-3)^2} \Rightarrow x = 0, \quad y(0) = -\frac{1}{3}$

stationary point at $(0, -\frac{1}{3})$ ✓ (coordinates)

b) Use the second derivative test to determine the nature of the stationary point.

$$y'' = -\frac{30x^2+30}{(x^2-3)^3}$$ (ClassPad) ✓ 2nd deriv.

$y''(0) = \frac{-30}{-27} = \frac{30}{27}$, $y''(0)>0$, positive, ∴ ✓ considers sign/concavity

concave up, Min T.P. ✓ at $(0, -\frac{1}{3})$ concludes

c) Are there any inflection points? Justify your answer using calculus.

for inflection points, 2nd derivative $=0$

Solve $y''(x) = 0$

✓ sets $y''=0$

$$-\frac{30x^2+30}{(x^2-3)^3} = 0$$

✓ indicates no solution

$$30x^2+30 = 0$$

No solution, answer is always >0

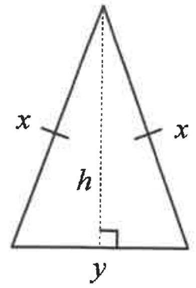
✓ reasons that there are no inflection points or change in concavity

* if students only reason using graph of $f(x)$, only give 1 mark

Question 8 **[3, 5 = 8 marks]** 3.1.16

An isosceles triangle has a perimeter of 60cm. If the two equal sides are labelled x, the third side y, and the perpendicular height h:

a) If it is known that $y = 60 - 2x$, show that $h = \sqrt{60x - 900}$



use pythagoras

$$h^2 + \left(\frac{y}{2}\right)^2 = x^2$$ ✓ uses pythag

$$h^2 + (30 - x)^2 = x^2$$

$$h^2 + 900 - 60x + x^2 = x^2$$ ✓ expands & rearranges

$$\therefore h = \sqrt{60x - 900} \quad //$$ ✓ concludes

b) Using calculus, determine the values of x, y and the area of the triangle if the area of the triangle is maximized.

$$A = \frac{1}{2} y h$$

$$A = \frac{1}{2}(60 - 2x)\sqrt{60x - 900}$$

$$\frac{dA}{dx} = \frac{-(3\sqrt{15}x - 60\sqrt{15})}{\sqrt{x - 15}}$$ ClassPad ✓ 1st derivative

max at $\frac{dA}{dx} = 0 \Rightarrow 0 = \frac{-\sqrt{15}x - 60\sqrt{15}}{\sqrt{x - 15}}$ ✓ sets $\frac{dA}{dx} = 0$

$x = 20$ ✓ solves for x

$y = 60 - 2(20) = 20$ ✓ solves for y

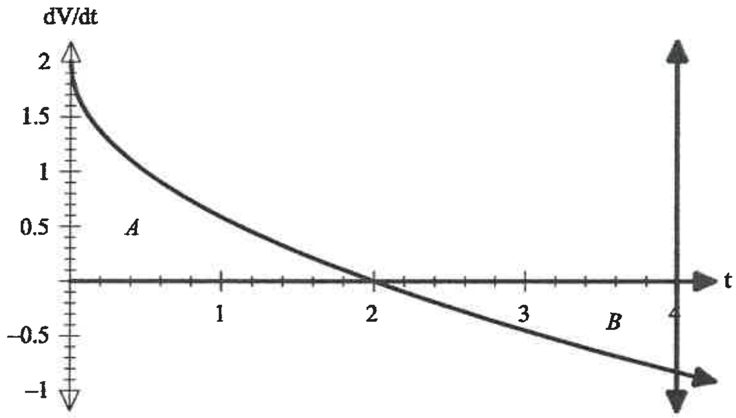
$$A(20) = \frac{1}{2}(\cancel{40}^{\,2}0)\sqrt{300}$$

$A_{max} = 100\sqrt{3}$ ~~units~~ cm^2 ✓ Area

≈ 173.21 ~~units~~ cm^2

Question 9 **[1, 2, 2, 2 = 7 marks]** 3.2.19 , 3.2.12

The instantaneous rate with which the amount of liquid, V litres, in a tank, changes with respect to time t minutes, is modelled by $\frac{dV}{dt} = -\sqrt{2t} + 2$. The sketch of $\frac{dV}{dt}$ against t is shown below.



a) Explain what happens in the tank after 2 minutes

✓ Liquid starts flowing out of the tank.
(Volume of water goes down)

b) Find the area of region A and interpret your answer

✓def integral $\int_0^2 (-\sqrt{2t}+2)\,dt = \frac{4}{3}$ L

✓interp. this is the net liquid increase in the tank from 0 to 2 minutes.

c) Find the area of region B and interpret your answer

✓def int. $\int_2^4 (-\sqrt{2t}+2)\,dt = -0.8758$ L

this amount is the net decrease in tank, from 2 to 4 minutes.

d) Find the amount of liquid in the tank after 4 minutes, if initially there were 16 litres in the tank.

$V(4) = V(0) + A + B$ ✓✓

$= 16L + \frac{4}{3}L - 0.8758L = 16.457\,L$